REPORT D[illegible]

AD-A259 739

Form Approved
OMB No. 0704-0188

(2)

Public reporting burden for this collection of information ... gathering and maintaining the data needed ... collection of information, including suggestions ... Davis Highway, Suite 1204, Arlington, VA 222[illegible] ... time for reviewing instructions, searching existing data sources, ... regarding this burden estimate or any other aspect of this ... for Information Operations and Reports, 1215 Jefferson ... Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank) | 2. REPORT DATE: October 1992 | 3. REPORT TYPE AND DATES COVERED: FINAL - 1 Oct 89 - 30 Sep 92

4. TITLE AND SUBTITLE

Studies of Baroclinic Flow

5. FUNDING NUMBERS

PE 61102F
PR 2310
TA CS
GR AFOSR-90-0009

6. AUTHOR(S)

Richard L. Pfeffer

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Geophysical Fluid Dynamics Institute
Florida State University
Tallahassee FL 32306

8. PERFORMING ORGANIZATION REPORT NUMBER

AFOSR-TR- 92 [illegible] 20

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Force Office of Scientific Research/NL
Building 410
Bolling AFB DC 20332-6448
Lt Col Stobie

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

30 PP

93-01512

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release;
distribution unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

During the three years of this grant, nine papers were published or accepted for publication in peer-reviewed journals. They covered the following topics: (1) potential vorticity dynamics, (2) baroclinic instability in the presence of forcing, (3) baroclinic wave structure and transitions, (4) the role of mountains in the evolution of mid-latitude synoptic disturbances, (5) complex principle component analysis, and (6) numerical techniques verses nonlinear analytic methods.

14. SUBJECT TERMS

15. NUMBER OF PAGES: 29

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
(U)	(U)	(U)	

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18

93 1 [illegible] 4

FINAL

TECHNICAL REPORT

Under
AFOSR 90-0009

Studies of Baroclinic Flow

by

Richard L. Pfeffer
Principal Investigator
Geophysical Fluid Dynamics Institute
Florida State University
Tallahassee, FL 32306

October 1992

I. INTRODUCTION

During the three years of this grant, 9 papers have been published or accepted for publication in quality scientific journals with high standards of review, one paper has been submitted and one more will be submitted for publication based on full or partial support of AFOSR 90-0009, including the work of two Ph.D. candidates. The following list also includes a paper presented at the AMS Conference and a report (partially supported by this grant) on a software package for solving a general second order parameter involved system of elliptic partial differential equations on a two dimensional domain with Dirichlet and/or Neuman boundary conditions. Other related research under AFOSR 89-0462 was reported in the University Research Initiative Final Report supported by that grant.

Low Frequency Oscillations in a Rotating Annulus with Topography, by P. Bernardet, A. Butet, M. Déqué, M. Ghil and R. L. Pfeffer; *J. Atmos. Sci.*, 47, 3023–3043, 1990.

Implementation of PLTMG Version 6.0 on SCRI4D VAX with IRIS 4000 Graphics, by Jian Yu, FSU publication no., FSU-SCRI-90T-124, August 27, 1990.

Effects of Wave-Wave and Wave-Mean Flow Interactions on the Evolution of a Baroclinic Wave, by A. Barcilon and T. Nathan; *Geophys. Astrophys. Fluid Dyn.*, 56, 59–79, 1991.

Asymmetric Ekman Dissipation, Sloping Boundaries and Linear Baroclinic Instability, by H. Weng and A. Barcilon; *Geophys. Astrophys. Fluid Dyn.*, 59, 1–24, 1991.

Reflection of Hydrostatic Mountain Waves from Spatially Nonuniform layers, by W. Blumen and A. Barcilon; *Geophys. Astrophys. Fluid Dyn.*, 58, 25–43, 1991.

EXSHALL: A Turkel-Zwas Explicit Large Time-Step FORTRAN Program for Solving the Shallow-Water Equations in Spherical Coordinates, by I. M. Navon and Jian Yu, *Computers in Geoscience*, 17, 1311–1343, 1991.

Potential Vorticity Index Vacillation in the 1978/79 Winter: Its Relation to Teleconnection Patterns, by H.-Y. Weng; *Q. J. R. Met. Soc.*, 118, 327–350, 1992.

A Comparison of the Impact of Two Time-Differencing Schemes on the NASA/GLAS Climate Model, by R. L. Pfeffer, I. M. Navon and X. Zou; *Mon. Wea. Rev.*, 120, 1381–1393, 1992.

The Effects of Viscous Asymmetry on Baroclinic Equilibrated States, by H.-Y. Weng and A. Barcilon; *Proceedings of the Eighth Conference on Atmospheric and Oceanic Waves and Stability*, October 1991.

Nonlinear Prediction, Chaos and Noise by J. B. Elsner and A. A. Tsonis, *Bulletin of the Amer. Meteor. Soc.*, 73, 49–59, 1992.

The Role of Potential Vorticity Forcing in Topographically Induced Instabilities by T. Nathan; *Trends in Atmospheric Sciences*, edited by E. Menan, in press, 1992.

Low Frequency Oscillations of Forced Barotropic Flow by T. Nathan and A. Barcilon; submitted to *J. Atmos. Sci.*, 1992.

Nonlinear Evolution of Baroclinic Waves in the Presence of Zonal Mean Periodic Forcing by T. Nathan; to be submitted to *Tellus*, 1992.

Section II contains a summary of accomplishments under this grant and section III contains abstracts of the above listed papers.

II. Summary of Accomplishments

1. PV Dynamics

The goal of this research was to explore the conjecture that an index based on the zonal mean potential vorticity (PV) gradient on an appropriate isentropic surface may exhibit a better relationship with the large-scale circulation, and therefore with some global teleconnections, than traditional zonal indices based on zonal wind or surface pressure. This conjecture was based on the knowledge that large-scale atmospheric processes are governed entirely by the conservation of potential vorticity and that all dynamical fields, including the vertical motion, can be derived from the potential vorticity field on isentropic surfaces. With this in mind, Dr. H. Y. Weng (*Quart. J. Roy. Meteor. Soc.*, **118**, 327–350, 1992) defined a PV index as the zonal mean mid-latitude PV gradient on the 300 K isentropic surface in the Northern Hemisphere and used ECMWF FGGE III-b data to investigate the evolution of that index and its relationship with teleconnection patterns of the 500-mb geopotential height anomaly. This index fluctuated with dominant periods of the order of two weeks and 30–50 days. It was found that the formation of blocking highs in Scandinavia, Greenland and the Pacific occurred at rather specific phases of the evolution of the PV index, especially as the flow transited from low to high values of this index. The blocking highs proceeded in an orderly fashion from Scandinavia to Greenland to the Pacific, suggesting that they are associated with Westward propagating planetary waves. When the study was repeated using a more conventional 500 mb zonal wind index, the results did not show such a clear relationship with the teleconnection patterns. Thus, Dr. Weng succeeded in developing a new and useful index.

2. Baroclinic Instability in the Presence of Forcing

The focus of this research was to understand the interactions between the stationary planetary scale and the transient synoptic scales of atmospheric behavior. To this end Drs. A. Barcilon and T. Nathan (*Geophys. Astrophys. Fluid Dyn.*, **56**, 59–79, 1991) studied the nonlinear evolution of a free baroclinic wave in the presence of slightly supercritical, vertically sheared zonal flow and a forced stationary wave field consisting of a single zonal scale and an arbitrary number of meridional harmonics. The analysis was carried out within the context of the two-layer baroclinic model. Using a multiple time scale analysis, Drs. Barcilon and Nathan found that the presence of the planetary-scale stationary wave induces zonal variations in the supercriticality which alters the growth rate and asymptotic equlibration of the baroclinic wave. This comes about in part by the direct interaction of the synoptic scale wave with the forced wave, and in part by the interaction

of the synoptic-scale wave with the zonal mean flow which has been altered by the stationary wave fluxes. When the stationary wave field is confined to the upper layer and consists of only the gravest cross-stream mode, this condition is favorable for the non-zero equilibration of the free wave. When it is confined to the lower layer, this condition is unfavorable for such equilibration.

3. Baroclinic Wave Structures and Transitions

This study was motivated by the need to resolve controversies in the meteorological literature concerning the effects of Ekman layer dissipation and sloping boundaries on baroclinic waves. For example, (*i*) Is a small amount of Ekman layer dissipation stabilize or destabilize baroclinic waves? (*ii*) If there is frictional instability for small dissipation, how small should the dissipation be? (*iii*) What are the differences between the effects of symmetric and asymmetric Ekman layer dissipation? (*iv*) What are the effects of traditional (more friction at the bottom than the top) and nontraditional (less friction at the bottom than at the top — for example, if there is strong gravity waves action at the jet stream level)? (*v*) How are these effects altered in the presence or absence of sloping boundaries? (*vi*) For easterly wind shear how do the stability results change? In an effort to answer these questions, Drs. Weng and Barcilon (*Geophys. Astrophys. Fluid Dyn.*, **59**, 1–24, 1991) first studied the effects of Ekman dissipation and sloping boundaries separately in a linear framework adding factors one-by-one to determine how each altered the solution. Then they examined the effects of nonlinear wave-mean flow and wave-wave interactions on baroclinic waves in the presence of viscous asymmetry (preprint of Eighth Conference on Atmospheric and Ocean Waves and Stability, Oct. 14-18, 1991, 134-137). Their main conclusions were as follows:

In the absence of sloping boundaries, symmetric Ekman layer dissipation has a stabilizing tendency for all waves. Asymmetric Ekman layer dissipation, on the other hand, destabilizes the waves by extending the unstable wave band toward both long and short waves and by increasing all growth rates in comparison with symmetric dissipation. It also makes the waves dispersive and their structure asymmetric about mid-depth.

In the presence of sloping boundaries wave dispersion increased and the wave propagation characteristics became sensitive to the sign of the shear and the sense of the viscous asymmetry. For westerly wind shear, the wave amplitude was reduced and the waves became more baroclinic in the lower atmosphere and barotropic above. The reverse wave true for easterly wind shear. The inclusion of nonlinearity did not affect these basic results.

4. Role of Mountains

The goal of this research was to ascertain the effects of topographically forced planetary waves on the genesis and evolution of mid-latitude synoptic disturbances. The approach was to investigate the linear and nonlinear stability of zonally varying basic flows within the context of the quasi-geostrophic barotropic and two-layer baroclinic models. The models were constructed so as to combine baroclinic, barotropic and topographic processes in the form of primary and secondary instabilities. The treatment of the nonlinear problem was done analytically using asymptotic expansions and multiple time scales.

To this end, Drs. Barcilon and Nathan first examined the nonlinear evolution of a free baroclinic wave in a two-layer atmosphere in the presence of slightly supercritical, vertically sheared zonal flow and a forced stationary wave consisting of a single zonal scale with an arbitrary number of meridional harmonics. Using a multiple time scale analysis, they showed that the stationary wave induces zonal variations in supercriticality which alters the growth rate and asymptotic equilibration of synoptic disturbances via two mechanisms. The first is the direct interaction of the synoptic disturbance with the forced wave (i.e., a wave-wave mechanism). The second is the interaction of the synoptic disturbance with the portion of the mean field that has been changed by the stationary wave fluxes (a wave-mean flow mechanism). These mechanisms oppose or augment each other depending on the amplitude and spatial structure of the stationary wave field. If it is confined to the lower layer, and consists only of the gravest cross stream mode, the free wave does not equilibrate to a finite amplitude. If it is confined to the upper layer, the free wave does equilibrate to a finite amplitude. The interaction of a synoptic wave with a stationary wave consisting of two or more meridional harmonics generates time dependent heat fluxes that vary with the period of the synoptic wave. If, however, the stationary wave field contains several sufficiently large amplitude meridional harmonics it can destroy the synoptic wave.

In a further study (*Trends in Atmos. Sci.*, 1992, in press), Dr. Nathan addressed the issue of how the combined influence of topographic and zonally varying potential vorticity (PV) forcing can influence low frequency motions in the atmosphere. In particular, he demonstrated analytically that such forcing can produce dramatic changes in the baroclinic topographic instability properties of the flow. The PV forcing can enhance, suppress or catalyze the topographic instability *depending on the phase* of the PV forcing with respect to the topography. These changes result from the alteration of the zonal mean flow produced by the interaction between the resonant wave and the PV forcing. Since the PV forcing in Dr. Nathan's model reflects the diabatic heating in the

atmosphere, a major conclusion of this study is that *land- sea heating contrasts may play a more important role in the dynamics of topographically induced instabilities than previously thought.*

Drs. Nathan and Barcilon also extended the Jin and Ghil (1990) barotropic model of topographically induced instabilities to include a steady, zonally varying vorticity source. In particular, they showed that, in contrast to the results obtained by JG, the vorticity forcing is capable of generating low frequency oscillations that remain strictly upstream of the topographic ridge. For sufficiently large vorticity forcing, the Hopf bifurcation is destroyed, irrespective of the relative phase of the forcings. In addition to the lower frequency oscillations obtained by JG, they also obtained solutions that exhibit two distinctly different types of regimes. One regime is a westward-propagating, low frequency oscillation that has the character of a free wave. The second regime is characterized by a quasi-steady state in which the wave ridge remains upstream of the topographic ridge. This two regime solution is quite robust and only occurs when the vorticity forcing augments the topographic forcing.

Finally, Drs. Barcilon and Blumen (*Geophys. Astrophys. Fluid Dyn.*, 58, 25–43, 1991) examined the formation of large-amplitude downslope wind storms such as those that occur to the lee of the Rockies. They found criteria on the vertical stratification of the atmosphere and cross-ridge wind variations that would cause internal gravity waves generated by an airstream passing over a mountain to amplify explosively. The cross-ridge wind variations produce significant differences in the surface wind in the lee of the mountain and in the surface drag compared to cases with no such variations.

5. Complex Principal Component Analysis

Low frequency oscillations were also studied by Dr. Pfeffer and his colleagues (Bernardet, et. al., *J. Atmos. Sci.*, 47, 3023–3043, 1990) using complex principal component analysis of data from rotating laboratory experiments in a thermally driven fluid over bottom topography. In these experiments the topography generated a planetary-scale stationary wave and the baroclinicity produced synoptic-scale traveling disturbances. The complex principal component analysis of data measured at two levels in the fluid showed the presence of the standing wave with a *baroclinic structure* which modulated the synoptic scale waves traveling over it. The analysis also revealed that the travelling wave energy was concentrated downstream of the topographic ridges, suggesting the mechanism by which storm tracks are maintained in the atmosphere — primarily by mountains rather than by land-sea temperature contrasts. Another significant result was that of *symmetry*

breaking. That is, although the topography consisted of two perfectly symmetric, sinusoidal mountains the wave trains downstream of one mountain were quite different from those downstream of the other suggesting that it is *not.* The different shapes of the Rockies and the Himalayas that decouple persistent anomalies in the Atlantic from those in the Pacific, but rather the nonlinear dynamics that lead to symmetry breaking even in simple laboratory experiments.

The data from another series of laboratory experiments was also used by Drs. Elsner and Tsonis to test methodologies of nonlinear prediction of chaotic time series.

6. Numerical Studies

Our numerical studies were motivated by the need to supplement nonlinear analytic studies with numerical studies and to understand the limitations of both. In this connection Dr. Navon and his student Jian Yu (*Computers in Geosciences*, 17, 1311–1343, 1991) developed and documented a computer program applying the Turkel-Zwas explicit large time-step scheme to a hemispheric barotropic model with constraint restoration of integral invariants of the shallow water equations. Mr. Yu (FSU-SCRI-90T-124, 1990) also implemented and documented the software package PLTMG for solving a general second order parameter involved system of elliptic partial differential equations on a two-dimensional domain with Dirichlet and/or Neuman boundary conditions. And Drs. Pfeffer, Navon and Zou studied the effects of using two different time-differencing schemes in the numerical integration of a global model. This last study revealed that significant differences in the solutions occur when only the time scheme is changed. This makes it imperative to carry out both analytical and numerical studies and to limit conclusions only to results that can be corroborated by both methodologies.

III. ABSTRACTS OF PAPERS

Contribution No. 299
Geophysical Fluid Dynamics Institute
Florida State University

Reprinted from JOURNAL OF THE ATMOSPHERIC SCIENCES, Vol. 47, No. 24, 15 December 1990
American Meteorological Society

Low-Frequency Oscillations in a Rotating Annulus with Topography

P. BERNARDET, A. BUTET AND M. DÉQUÉ

Centre National de Recherches Météorologiques, EERM/DMN, Toulouse, France

M. GHIL

Climate Dynamics Center, Department of Atmospheric Sciences and Institute of Geophysics and Planetary Physics, University of California at Los Angeles, California

R. L. PFEFFER

Geophysical Fluid Dynamics Institute and Department of Meteorology, Florida State University, Tallahassee, Florida

(Manuscript received 19 February 1990, in final form 13 July 1990)

ABSTRACT

Experiments were performed in a rotating, differentially heated annulus, with and without bottom topography of azimuthal wavenumber 2. Both water and a viscous glycerol–water mixture were used as a working fluid. In one series of experiments, measurements of azimuthal velocity u were carried out by Doppler-laser velocimetry at midradius and at $^1/_3$ and $^2/_3$ depth. In the other, temperature measurements were made by a set of thermistors at three different heights and three different radii. Results were analyzed by Fourier transformation, separately in space and in time, and in terms of complex empirical orthogonal functions (CEOFs).

In the experiments with topography, a standing wave 2 is generated, with larger amplitude at the upper level and a tilted wave structure. The two leading CEOFs contain a very large fraction of the variance, and give an excellent picture of the spatial modulation of the traveling baroclinic waves. The dominant baroclinic wave has azimuthal wavenumber 4, 5 or 6, according to the nondimensional parameters of the given experiment, and pronounced sidebands due to the topography. The modulation of this wave is such that its largest amplitude occurs at the lower level upstream of the two topographic ridges. At the upper level, the modulation is weaker, with the maximum wave amplitude located downstream of the ridges. Partial decoupling of the two wave trains attached to the two ridges is evident in one experiment.

Low-frequency vacillation of the entire flow pattern is apparent; this vacillation has a period of about 50 annulus rotations in the viscous mixture. The possible relevance of this topographically induced vacillation to the extratropical 30–60 day oscillation is discussed.

1. Introduction and motivation

Meteorology is mainly an observational, rather than an experimental science. Still, rotating annulus experiments represent a powerful paradigm for the general circulation of the atmosphere's midlatitudes, including its variability (Lorenz 1967, Ch. VI; Ghil and Childress 1987, Ch. 5). Detailed comparisons between the fluid-flow phenomena and their causal mechanisms in the annulus, on the one hand, and analogous phenomena in the atmosphere, on the other, are often difficult and sometimes controversial (Wallace and Hsu 1985). But the rotating, differentially heated annulus provides a fluid environment for which parameters can be changed, measurements taken and observed phenomena explained with greater ease than for the atmosphere itself. Whether these phenomena, and their explanations, are the same as in the atmosphere has to be decided on a case-by-case basis. The dynamically interesting observations and explanations, however, arising from the annulus experiments are an important source of ideas in the study of large-scale atmospheric motions.

In studies of the atmosphere, low-frequency variability occupies an increasingly important part (Hoskins and Pearce 1983; Branstator et al. 1988). In particular, intraseasonal oscillations with periods of roughly 25 days (Branstator 1987; Ghil and Mo 1990) and 40 days (Weickmann et al. 1985; Lau and Phillips 1986; Ghil and Mo 1990) in the Northern Hemisphere (NH) extratropics have attracted some attention recently. Theoretical explanations of the extratropical NH 30–60 day oscillation have tended to fall into two categories. First, these oscillations have been seen as an essentially linear response to tropical forcing with the same range of periods (see, for example, Hsu et al. 1990, and references therein). Second, they have been

Corresponding author address: Dr. R. L. Pfeffer, Geophysical Fluid Dynamics Institute, Florida State University, Tallahassee, FL 32306-3017.

Geophys. Astrophys. Fluid Dynamics. Vol. 56. pp. 59–79
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Contribution No. 303
Geophysical Fluid Dynamics Institute
Florida State University

EFFECTS OF WAVE–WAVE AND WAVE–MEAN FLOW INTERACTIONS ON THE EVOLUTION OF A BAROCLINIC WAVE

ALBERT BARCILON

Department of Meteorology and Geophysical Fluid Dynamics Institute. Florida State University, Tallahassee, FL 32306, USA

TERRENCE R. NATHAN

Department of Land, Air, and Water Resources, University of California, Davis, CA 95616, USA

(Received 11 June 1990; in final form 18 September 1990)

The weakly nonlinear evolution of a free baroclinic wave in the presence of slightly supercritical, vertically sheared zonal flow and a forced stationary wave field that consists of a single zonal scale and an arbitrary number of meridional harmonics is examined within the context of the conventional two-layer model. The presence of the (planetary-scale) stationary wave introduces zonal variations in the supercriticality and is shown to alter the growth rate and asymptotic equilibrium of the (synoptic-scale) baroclinic wave via two distinct mechanisms: The first is due to the direct interaction of the stationary wave with the shorter synoptic wave (wave–wave mechanism), and the second is due to the interaction of the synoptic wave with that portion of the mean field that is corrected by the zonally rectified stationary wave fluxes (wave–mean mechanism). These mechanisms can oppose or augment each other depending on the amplitude and spatial structure of the stationary wave field. If the stationary wave field is confined primarily to the upper (lower) layer and consists of only the gravest cross-stream mode, conditions are favorable (unfavorable) for nonzero equilibrium of the free wave.

In addition to the time dependent heat flux generated by baroclinic growth of the free wave, its interaction with a stationary wave field consisting of two or more meridional harmonics generates time dependent heat fluxes that vary with period of the free wave. However, if the stationary wave field contains several meridional harmonics of sufficiently large amplitude, the free baroclinic wave is destroyed.

KEY WORDS: Baroclinic instability, stationary waves, wave–wave and wave–mean flow interactions.

1. INTRODUCTION

A characteristic signature of geophysical fluid systems is the presence of several mutually interacting scales of motion (e.g., Colucci *et al.*, 1981; Li *et al.*, 1986; Pfeffer *et al.*, 1990). Here we focus on the finite-amplitude interactions between *two* scales of motion that characterize the large-scale circulation of the atmosphere: the stationary planetary-scales and the transient synoptic-scales.

*Contribution No. 303 of the Geophysical Fluid Dynamics Institute, Florida State University, Tallahassee, FL.

Contribution No. 312
Geophysical Fluid Dynamics Institute
Florida State University

Geophys. Astrophys. Fluid Dynamics, Vol. 59, pp. 1-24
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ASYMMETRIC EKMAN DISSIPATION, SLOPING BOUNDARIES AND LINEAR BAROCLINIC INSTABILITY

HENG-YI WENG and ALBERT BARCILON*

Geophysical Fluid Dynamics Institute, Florida State University, Tallahassee, FL 32306, USA

(Received 12 December 1990; in final form 19 April 1991)

The effects of symmetric and asymmetric Ekman dissipation on baroclinic instability, phase speed and wave structure in a linear Eady-like channel model with/without oppositely sloping boundaries are examined. In the absence of sloping boundaries, symmetric Ekman dissipation has a stabilizing tendency for all waves. When the Ekman layers are asymmetric, the viscous asymmetry destabilizes waves by (i) extending the unstable waveband toward both long and short waves, and (ii) increasing their growth rates, compared with viscous symmetric case. When the asymmetric dissipation is very small, the destabilization may result in a frictional instability for short waves which are stable in inviscid case. The viscous asymmetry makes waves dispersive and their structure asymmetric about mid-depth. However, the sense of the viscous asymmetry and the sign of shear do not affect the instability, but do modify the direction of phase propagation and the shape of the wave structure.

With asymmetric Ekman dissipation and sloping boundaries, frictional instability is a mixture of three mechanisms: (i) symmetric dissipation in the presence of sloping boundaries; (ii) viscous asymmetry in the absence of sloping boundaries; and (iii) the sense of viscous asymmetry in the presence of sloping boundaries, which is sensitive to the wavenumber and the sign of shear. The slope renders the phase speed more dispersive and sensitive to the sign of shear and the sense of the viscous asymmetry. For westerly shear, the slope reduces (increases) the wave amplitude and makes the wave more baroclinic (barotropic) in the lower (upper) level. For a given wavenumber, easterly shear dynamics may be deduced from westerly shear dynamics by a proper change of the sense of the viscous asymmetry.

KEY WORDS: Baroclinic waves, Ekman dissipation, sloping boundaries.

1. INTRODUCTION

Because of its linear and nonlinear analytical simplicity, the Eady (1949) model of baroclinic instability has served as a "workhorse" to test various aspects of that instability. Yet, we feel that the various possible alternatives offered by this model, such as asymmetric Ekman dissipation and sloping boundaries in particular, could stand a thorough re-examination which we offer below to lay the foundation before proceeding with nonlinear studies.

Using a two-layer β-plane model, Holopainen (1961) considered a *single* Ekman layer at the lower boundary and found that the effect of friction reduces the amplitude of the perturbation, except within two narrow wavebands adjacent to the two inviscid

Contribution No. 312 of the Geophysical Fluid Dynamics Institute, Florida State University, Tallahassee, FL.

* Also at the Department of Meteorology.

Geophys. Astrophys. Fluid Dynamics, Vol. 58, pp. 25–43
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REFLECTION OF HYDROSTATIC MOUNTAIN WAVES FROM SPATIALLY NONUNIFORM LAYERS

WILLIAM BLUMEN

Astrophysical, Planetary and Atmospheric Sciences Department, University of Colorado, Boulder, CO 80309, USA

ALBERT BARCILON

*Department of Meteorology and Geophysical Fluid Dynamics Institute, Florida State University, Tallahassee, FL 32306, USA**

(Received 7 August 1990; in final form 5 September 1990)

The steady, hydrostatic, inviscid, Boussinesq flow of a stably stratified fluid over a bell-shaped ridge is investigated within the framework of a linear model. The three layer model atmosphere introduced is such that the Brunt-Väisälä frequency is constant in each layer but the interfaces of the middle layer are allowed to vary gently in the cross-ridge direction. In essence, the model can be tuned in both *vertical* and *horizontal* directions. These cross-ridge variations can produce significant differences in both the cross-ridge surface wind and the surface drag compared to the response obtained by use of a horizontally uniform reflecting layer. These changes are sensitive to both the vertical location of the middle layer and to the slope of its lower interface at the ridge crest. Many of these features are explained by means of a conventional layered-model analysis.

KEY WORDS: Mountain waves, stratified flow, Boussinesq fluid.

1. INTRODUCTION

Typically, the static stability, expressed by the Brunt–Väisälä frequency, undergoes considerable variability in the vertical direction. Figure 1 shows that the static stability, N^2, exhibits variability over scales of 1–2 km in the troposphere. Similar features are retained between the two stations, as in the region between 6 and 8 km in panel b. In other cases, however, the apparent layering of the static stability may be localized in the vicinity of each station, and/or the same features may ascend or descend in altitude between GCT and DEN. This latter possibility, which may characterize the features in panel a, is examined in the present study.

We consider the dynamics of hydrostatic mountain waves in the presence of static stability variations in the cross-ridge direction. Such states will consist of regions bounded by x-dependent interfaces across which the Scorer parameter changes abruptly, remaining constant on either sides of these interfaces. These

*Contribution no. 304 of the Geophysical Fluid Dynamics Institute, Florida State University, Tallahassee, Florida.

Q. J. R. Meteorol. Soc. (1992), **118**, pp. 327–350 551.511.2:551.513.7

Potential vorticity index vacillation in the 1978/79 winter: Its relation to teleconnection patterns†

By HENG-YI WENG

Geophysical Fluid Dynamics Institute, Florida State University, Tallahassee, FL 32306, USA

Contribution No. 313
Geophysical Fluid Dynamics Institute
Florida State University

(Received 1 November 1990; revised 7 May 1991)

SUMMARY

A potential vorticity (PV) index is defined as a measure of the zonally averaged, mid-latitude PV gradient on the 300 K isentropic surface in the northern hemisphere, and use is made of the 1978/79 winter ECMWF FGGE III-b data set to study the evolution of that index and its relation to teleconnection patterns of 500 mb geopotential-height anomaly. This index vacillates with a dominant period of about two weeks. For some time lags, the isoline patterns of the cross-correlation function between the PV index and 500 mb geopotential-height anomalies are similar to some teleconnection patterns studied by several investigators using large climatological data sets. Scandinavia, Greenland and the Pacific are important centres of 'action' in the northern hemisphere in the 1978/79 winter. The formation of blocking highs at these locations occurs at rather specific phases during the vacillatory evolution of the PV index, especially as the flow transits from a low to a high PV index.

When the above study is duplicated using a zonal mean wind gradient at 500 mb as a zonal index, the results do not reflect the teleconnection patterns as crisply as the PV index does for the FGGE year.

1. INTRODUCTION

Using the European Centre for Medium-range Weather Forecast (ECMWF) First GARP Global Experiment (FGGE) III-b data from 1 February to 15 May 1979, Kidson (1985) defined a zonal index in terms of the zonal mean wind difference between two latitude belts at 500 mb, one centred at 60°N and the other centred at 39.25°N. His spectral analysis showed that the 20- to 30-day cycle was most prominent near 40°N–60°N. He found that a low-index flow related to the presence of persistent cut-off lows at 500 mb off the coast of Newfoundland, with a warm ridge or a cut-off high in the vicinity of Iceland; while a high-index flow corresponded to a zonal flow at high latitudes, with slow-moving patterns over the Atlantic at lower latitudes and to the south of the main jet. Although his correlation pattern between the mean-sea-level pressure and the zonal index was broadly similar to the North Atlantic Oscillation (NAO) pattern found in climate data, some differences were apparent, particularly the absence of any significant variations in the Pacific, which, if present, should be related to the North Pacific Oscillation (NPO) or/and the Pacific/North American (PNA) patterns.

Teleconnections and seesaws have been found in climate studies by many investigators. For example, van Loon and Rogers (1978, hereafter referred to as vLR) found the Greenland–Northern Europe seesaw pattern, where winter temperatures are below normal over northern Europe while being above normal over Greenland and the Canadian arctic; and *vice versa*. They also found that sea-level-pressure anomalies over most of the northern hemisphere are associated with this regional seesaw in temperature. Wallace and Gutzler (1981, hereafter referred to as WG) summarized, based on existing literature, four teleconnection patterns: the NAO, the NPO, the PNA, and a zonally symmetric seesaw pattern found by Lorenz (1951) in the zonally averaged sea-level-pressure field. Since the strength and location of these patterns vary from winter to winter, the main climatic teleconnection patterns may differ, depending on the data set

† Contribution No. 313 of the Geophysical Fluid Dynamics Institute.

Computers & Geosciences Vol. 17, No. 9, pp. 1311-1343, 1991
Printed in Great Britain

0098-3004/91 $3.00 + 0.00
Pergamon Press plc

EXSHALL: A TURKEL–ZWAS EXPLICIT LARGE TIME-STEP FORTRAN PROGRAM FOR SOLVING THE SHALLOW-WATER EQUATIONS IN SPHERICAL COORDINATES*

I. M. NAVON[1,2] and JIAN YU[1]

[1]Department of Mathematics, and [2]Supercomputer Computations Research Institute, Florida State University, Tallahassee, FL 32306, U.S.A.

(Received 3 July 1990; revised 15 August 1991)

Abstract—A FORTRAN computer program is presented and documented applying the Turkel–Zwas explicit large time-step scheme to a hemispheric barotropic model with constraint restoration of integral invariants of the shallow-water equations. We then proceed to detail the algorithms embodied in the code EXSHALL in this paper, particularly algorithms related to the efficiency and stability of T–Z scheme and the quadratic constraint restoration method which is based on a variational approach. In particular we provide details about the high-latitude filtering, Shapiro filtering, and Robert filtering algorithms used in the code. We explain in detail the various subroutines in the EXSHALL code with emphasis on algorithms implemented in the code and present the flowcharts of some major subroutines. Finally, we provide a visual example illustrating a 4-day run using real initial data, along with a sample printout and graphic isoline contours of the height field and velocity fields.

Key Words: Shallow-water equations, Spherical coordinates, Explicit finite-differences, Constraint restoration, Filtering techniques.

INTRODUCTION

Recently, a considerable amount of work has been dedicated and aimed at efficient integration of shallow-water equations in view of using these methods in numerical weather prediction models. In order to achieve computational accuracy and efficiency, most methods are concerned with the different time-scale of the advection and the gravity-inertia terms in the shallow-water equations model separately. Semi-implicit schemes (Robert, 1979; Burridge, 1975) and split-explicit schemes (Magazenkov, Shvets, and Shneyerov, 1971; Gadd, 1978a, 1978b) are examples of those methods. In the split-explicit schemes, a substantial computational economy is achieved when compared to usual explicit time integration schemes.

Turkel and Zwas (1979) proposed a space-splitting rather than a time-splitting algorithm for the explicit integration of the shallow-water equations. Their method is based on the fact that the fast gravity-inertia waves contain only a small fraction of the total available energy and therefore these waves can be calculated with a lower accuracy than the slow Rossby waves, that is on a coarser mesh. An application of the T–Z space split-explicit integration schemes with real initial data is presented and discussed by Navon and de Villiers (1987) and its properties are discussed further in Neta and Navon (1989). A linear transfer function analysis of the shallow-water equations in spherical coordinates for the Turkel–Zwas explicit large time-step scheme was carried out by Neta, Navon, and Yu (1990).

The purpose of this paper is to present a practical FORTRAN code, EXSHALL, which implements the T–Z explicit large time-step scheme for the shallow-water equations in spherical coordinates along with constraint restoration methods for enforcing a posteriori conservation of the integral invariants of the shallow-water equations. The computer program is explained in detail in connection with the various algorithms implemented in the code EXSHALL. This paper can be used as a user's guide to the program EXSHALL both in providing a brief description of the theory as well as detailed programming implementation.

We present here the T–Z scheme for the shallow-water equations in spherical coordinates and its related algorithmic background. Various filters used with T–Z scheme and which impact on its stability also are presented; a detailed description of the various subroutines in the code EXSHALL is presented; and a typical example of a 4-day run with the program EXSHALL is presented along with graphical output. Finally, in the Appendix the commented and documented FORTRAN source listing the code of the program EXSHALL is attached.

*Geophysical Fluid Dynamic Institute Contribution No. 329.

Reprinted from MONTHLY WEATHER REVIEW, Vol. 120, No. 7, July 1992
American Meteorological Society

Contribution No. 328
Geophysical Fluid Dynamics Institute
Florida State University

A Comparison of the Impact of Two Time-differencing Schemes on the NASA–GLAS Climate Model*

RICHARD L. PFEFFER

Geophysical Fluid Dynamics Institute and Department of Meteorology, Florida State University, Tallahassee, Florida

I. M. NAVON

Department of Mathematics, Supercomputer Computations Research Institute and Geophysical Fluid Dynamics Institute, Florida State University, Tallahassee, Florida

XIAOLEI ZOU

Supercomputer Computations Research Institute, Florida State University, Tallahassee, Florida

(Manuscript received 27 February 1991, in final form 25 October 1991)

ABSTRACT

This paper presents evidence of the sensitivity of a general circulation model (GCM) to the time-differencing scheme employed when the physical parameterizations and space discretization are not changed. For this purpose, two time-marching schemes—the leapfrog and the Matsuno schemes—are analyzed and tested on the National Aeronautics and Space Administration–Goddard Laboratory for Atmospheric Studies (NASA–GLAS) fourth-order GCM in terms of the stability and behavior of 2-month-averaged fields. Linear analysis suggests that Rossby waves are slightly damped and slightly accelerated when the Matsuno scheme is used and that these effects are scale selective, being smallest for the longest waves. It also suggests that such waves are accelerated less and are not damped when the leapfrog scheme is used. An empirical orthogonal function analysis of the meridional component of velocity at 46°N, keeping at least 70% of the variance, reveals less shortwave activity in the numerical solution with the Matsuno scheme but does not lend support to the conclusion that the waves are accelerated less in the solution with the leapfrog scheme.

The two-dimensional Eliassen–Palm (E–P) flux divergence and the eddy-induced mean meridional circulation are found to be stronger in the simulation with the leapfrog time-differencing scheme than in the one with the Matsuno scheme, suggesting that the transient-wave activity is damped by the Matsuno scheme. On the other hand, the three-dimensional stationary-wave activity flux in the Northern Hemisphere simulated with the Matsuno scheme is more intense than that simulated with the leapfrog scheme, indicating that the stationary waves are more robust in the integration with the Matsuno scheme.

The GCM precipitation when integrated with the leapfrog scheme is much more intense over the tropical western Pacific and the northeastern Pacific and less intense over the western North Atlantic Ocean. The kinetic energy of waves with wavenumber greater than 9 simulated by the Matsuno scheme is consistently smaller than that obtained by the leapfrog scheme. These results give evidence that climate simulations are sensitive not only to physical parameterizations of subgrid-scale processes but also to the numerical methodology employed.

1. Introduction

General circulation models (GCMs) are the most elaborate of a hierarchy of mathematical models used in the study of climate. A general review of atmospheric GCMs presented by Simmons and Bengtsson (1984) emphasizes the importance of physical processes in determining the behavior of climatic systems. GCMs have been used for seasonal simulation (Shukla et al. 1981) as well as for studying climate variability on time scales of a month and upward. In particular, studies have been conducted to determine the sensitivity of atmospheric GCMs to changes in physical mechanisms such as surface albedo (Charney 1975), sea-ice limits in the Arctic (Herman and Johnson 1978), low-frequency variability (Charney and Shukla 1981), and inadequate orographic effects (Wallace et al. 1983).

Reviews of the numerical techniques used in numerical weather prediction models and GCMs have been presented by Mesinger and Arakawa (1976) and Kasahara (1979). Numerical experiments in which the earth's climate is simulated using different numerical schemes can play a valuable role in clarifying the nature

* Contribution No. 328 of the Geophysical Fluid Dynamics Institute

Corresponding author address: Professor Richard L. Pfeffer, Geophysical Fluid Dynamics Institute, Florida State University, Tallahassee, FL 32306.

Contribution No. 323
Geophysical Fluid Dynamics Institute
Florida State University

THE EFFECTS OF VISCOUS ASYMMETRY ON BAROCLINIC EQUILIBRATED STATES

Hengyi Weng and Albert Barcilon*

Geophysical Fluid Dynamics Institute
Florida State University
Tallahassee, Florida

1. INTRODUCTION

The effect of asymmetric Ekman dissipation on baroclinic waves has been studied in linear (e.g., Holopainen, 1961) and nonlinear (e.g., Mak, 1987) frameworks. Weng and Barcilon (1991, WB) compared the effects of asymmetric Ekman dissipation, with/without sloping boundaries, on *linear* baroclinic waves in an Eady-like model. They found that when the asymmetric Ekman layers are horizontal, viscous asymmetry destabilizes waves by (i) extending the unstable waveband toward both long and short waves, and (ii) increasing their growth rates, when compared with the symmetric case. When the asymmetric dissipation is very small, the destabilization may result in a frictional instability for short waves which are stable in the inviscid case. Viscous asymmetry makes waves dispersive and their structures are asymmetric about mid-depth. However, the *sense* of viscous asymmetry and the *sign* of the vertical shear do not affect the instability, but do modify the direction of phase propagation and the shape of the wave structure. When the asymmetric Ekman layers are oppositely sloping, the effect of viscous asymmetry is modified by the slope. The frictional instability is a mixture of three mechanisms: (i) symmetric dissipation in the presence of sloping boundaries; (ii) viscous asymmetry in the absence of sloping boundaries; and (iii) the sense of viscous asymmetry in the presence of sloping boundaries, which is sensitive to the wavenumber and the sign of the shear. Thus, the slope renders the phase speed more dispersive and sensitive to the *sign* of the shear and the *sense* of viscous asymmetry. For a given wavenumber, easterly shear dynamics may be deduced from westerly shear dynamics by interchanging the asymmetric top and bottom Ekman layers.

Mak (1987) in a two-layer β-plane model found that a vertical asymmetry in dissipation *qualitatively* influences the equilibration of nonlinear quasi-geostrophic unstable baroclinic waves. Especially, he found that the *sense* of this asymmetry is a *crucial* factor that dictates whether the equilibrated state is generally a steady multiple wave state or a vacillation. It is uncertain whether Mak's results are due to the sole effect of viscous asymmetry or the combined effect of that asymmetry with β. To settle this question, we isolate the effect of viscous asymmetry on nonlinear waves from those due to β and other causes. The present work may play the role of a bridge between our previous linear analyses and future, more realistic, nonlinear analyses.

* Also in the Department of Meteorology, FSU

2. THE MODEL

We use the model of Weng (1990), with three zonal wavenumbers, each having two meridional modes, in wave field and four modes in mean flow correction (mfc). It is an Eady-like channel model having asymmetric Ekman layers at flat top and bottom; periodicity applies in the along channel direction. The nondimensional streamfunction is written as

$$\phi = \underset{\text{(basic)}}{-\lambda y z} + \underset{\text{(mfc)}}{\bar{\phi}} + \underset{\text{(wave)}}{\tilde{\phi}} \tag{1}$$

The nondimensional governing equation and boundary conditions are

$$\left.\begin{aligned} &\tilde{\phi}_{zz} + S\nabla^2\tilde{\phi} = 0, \\ &\tilde{\phi}_x = 0, \qquad \text{at } y = 0, 1, \\ &\tilde{\phi}_{zt} + z\lambda\tilde{\phi}_{xz} - \lambda\tilde{\phi}_x - 2z\gamma\delta\nabla^2\tilde{\phi} + J(\tilde{\phi}, \tilde{\phi}_z) - \overline{J(\tilde{\phi}, \tilde{\phi}_z)} + \\ &\qquad + J(\bar{\phi}, \tilde{\phi}_z) + J(\tilde{\phi}, \bar{\phi}_z) = 0, \text{ at } = \pm 1/2, \end{aligned}\right\} \tag{2}$$

and

$$\left.\begin{aligned} &\bar{\phi}_{zz} + S\bar{\phi}_{yy} = 0, \\ &\bar{\phi}_{yt} = 0, \qquad \text{at } y = 0, 1, \\ &\bar{\phi}_{zt} - 2z\gamma\delta\bar{\phi}_{yy} + \overline{J(\tilde{\phi}, \tilde{\phi}_z)} = 0, \qquad \text{at } z = \pm 1/2, \end{aligned}\right\} \tag{3}$$

where $\overline{(\ )}$ stands for a zonal average operator. The wave solution is

$$\tilde{\phi} = \sum_{l=1}^{2}\sum_{n=1}^{3} \Phi_{nl} e^{inx} \sin(l\pi y) + \text{c.c.} \tag{4}$$

where

$$\Phi_{nl} = A_{nl}\cosh(2\mu_{nl}z) + B_{nl}\sinh(2\mu_{nl}z) \tag{5}$$

and c.c. denotes "complex conjugate". The mean flow correction is

$$\bar{u} = -\bar{\phi}_y = \sum_{p=1}^{4}[M_p\cosh(2\omega_p z) + N_p\sinh(2\omega_p z)]\sin(p\pi y). \tag{6}$$

The nondimensional parameters and the notations in (1) – (6), as well as those used in this paper, are listed in Table 1.

Nonlinear Prediction, Chaos, and Noise

J. B. Elsner* and A. A. Tsonis*

Abstract

We present a brief overview of some new methodologies for making predictions on time-series data. These ideas stem from two rapidly growing fields: nonlinear dynamics (chaos) theory and parallel distributed processing. Examples are presented that show the usefulness of such methods in making short-term predictions. It is suggested that such methodologies are capable of distinguishing between chaos and noise. Implications of these ideas and methods in the study of weather and climate are discussed.

1. Introduction

One of the basic tenets of science is making predictions. If we know previous behavior, how can we predict future behavior? The approach in modern meteorology, like many sciences, requires two steps: construct a model based on theoretical considerations and use measured data as initial input. Since many of the underlying theoretical principles in meteorology are known, model construction has been and continues to be a primary area of research for meteorologists.

Today's numerical weather-prediction models for forecasting tomorrow's weather (also for climate prediction) solve a set of partial differential equations describing fluid flow over a rotating globe. The problem in prediction may not lie here. However, as was stated by Thompson (1957), significant problems may arise with the second step, where measured data are used as initial input to the model. Correct specification of initial state demands the measurements of variables in a three-dimensional volume. Routine measurements of relevant variables are taken at widely spaced locations providing only a discrete initial state. The spatially continuous differential equations simply cannot operate on discrete initial input (Farmer and Sidorowich 1987). Because of this inherent forecasting limitation in fluid-flow problems, we are motivated to try other approaches.

One class of alternative approaches is to build models directly from the available data. For these methods, the data, given as a time series, are usually considered as a single realization of a continuous random process (see, e.g., Pandit and Yu 1983). As Farmer and Sidorowich (1987) point out, this is appropriate when the randomness is a result of complex interactions involving many independent and irreducible degrees of freedom. Although linear methods of analyzing time series from weather and climate processes have had some success, especially in regard to relating cause and effect to physical phenomena, their predictive power is limited. The predictive limitation of linear methods is perhaps related to their inability to model feedback dynamics of the weather and climate systems (Farmer and Sidorowich 1988; hereafter referred to as FS88).

In the last decade, advances in the theory of dynamical systems have demonstrated the existence of dissipative systems whose trajectories that depict their asymptotic final states are not confined in limit cycles (periodic evolutions) or tori (quasi-periodic evolutions), but in sets of the total available phase space, which are not topological. These sets are fractal sets and are often called strange attractors. The corresponding dynamical systems are called chaotic systems and their trajectories never repeat. Thus, their evolution is aperiodic but completely deterministic. Because the evolution is aperiodic, any signal measured from a chaotic dynamical system "looks" quite irregular and exhibits frequency spectra with energy at all frequencies (broadband spectra) similar to those of random signals (see Tsonis and Elsner 1989 for a discussion of chaos and weather). Another important property of chaotic dynamical systems and their strange attractors is the divergence of initially nearby trajectories. Due to the action of the attractor, the evolution of the system from two (or more) nearby initial conditions will soon become quite different. Since the measurement of any initial condition is subject to some error, such a property imposes limits on long-term prediction. Nevertheless, for a short time, nearby trajectories may not diverge significantly, and thus, even though each evolution might be quite complex, knowledge of the dynamics and especially of the structure of the attractor (e.g., dimensions, Lyapunov exponents) may prove beneficial to the goal of short-term predictions.

For a system containing many irreducible degrees of freedom, the linear statistical approach is probably as good as any and may even be optimal (FS88). If, however, the irregular behavior is a result of low-

*Department of Meteorology, Florida State University, Tallahassee, FL 32306

*Department of Geosciences, University of Wisconsin-Milwaukee, Milwaukee, WI 53201

dimensional chaos, nonlinear models ought to be able to do much better at prediction than simple linear models. In fact, since chaos does not occur unless the system is to some extent nonlinear, nonlinear models are necessary to approximate chaotic dynamics.

The purpose of the present paper is to outline some recent advances in modeling time series and to demonstrate, through the application of a particular technique, their usefulness in making short-term predictions over standard autoregressive models. The paper is not intended to be definitive; rather, it serves as an interim report on time-series modeling efforts currently being explored in the physics and applied mathematical communities. No attempt is made to sort out the particular advantages and disadvantages of the various methods mentioned. More details concerning particular methods and applications are given in some of the references provided.

The problems of weather and climate forecasting offer a unique arena for testing and developing nonlinear prediction algorithms, not only because current numerical weather-prediction models are limited to some extent in their prediction capabilities, but because long-term reliable observational records have recently been made available for climate research. Diagnostic studies with these data utilizing nonlinear prediction schemes are a required step in the direction of understanding and quantifying the complexity of the global weather and climate systems.

2. Nonlinear prediction

The term "nonlinear prediction" covers a broad spectrum of methodologies. Our focus here is on dynamical state-space models. The two components of such models, determinism and state-space representation, can be considered separately. In fact, more familiar in meteorology are two variants of these models containing one or the other component. The analog method, suggested by Lorenz (1969), while not strictly limited to time-series data, is essentially a deterministic non-state-space model. It is based on the idea of finding a historical weather pattern (analog) that closely resembles the current weather. The evolution of the historical analog provides a model for the evolution of the present weather. Although the method, as tried by Lorenz, was successful in estimating practical limits on atmospheric predictability, it was largely unsuccessful for operational forecasting due to the lack of an adequate history of large-scale weather patterns. With longer data archives now available and a focus on limited areas, the analog method has reemerged and appears to hold promise for weather forecasting (e.g., Van den Dool 1989; Toth 1989).

Another class of related nonlinear models are the threshold autoregressive (AR) models (Tong and Lim 1980; Tong 1983). These models rely on a state-space representation but are essentially statistical, having a deterministic component limited to a single variable. While this is a considerable improvement (for modeling chaos) over the strictly statistical linear representation of the traditional AR models, it may not provide enough nonlinearity for geophysical signals in general. Recently, Zwiers and von Storch (1990) have shown that such models are quite useful for modeling the Southern Oscillation.

Building a dynamical state-space model from time-series data requires two steps: finding an appropriate state-space reconstruction, and then choosing a nonlinear representation that maps visited regions into regions not yet visited in the reconstructed space. The state space can be replaced by the phase space using the method of delays. This is done by taking a scalar time series $x(t_i)$ and its successive time shifts as coordinates of a new vector time series given by

$$X(t_i) = \{x(t_i), x(t_i+\tau), \ldots, x(t_i + (n-1)\tau)\}, \quad (1)$$

where n is the dimension of the vector $X(t_i)$ and τ is a time delay taken to be some suitable multiple of the sampling time Δt (see Packard et al. 1980; Takens 1981). Thus, for an n-dimensional phase space, a "cloud" of points will be generated. From this cloud the various dimensions and exponents can be calculated. The proper choice of τ to obtain a suitable reconstruction has been the subject of considerable debate. In principle, τ can be any length. However, if it is too small, then, in general, $x(t_i)$ will be nearly equal to $x(t_i + \tau)$ and not enough separation will exist between the chosen coordinates. If the dynamics take place on an attractor of dimension N, then it is necessary for determinism that $n \geq N$ (i.e., the attractor must be embedded in at least its dimension, otherwise it fills the embedding space, thus behaving like a random process) (FS88). For proper reconstructions, Takens (1981) showed that $n \sim 2N + 1$ is sufficient at least in principle.

There are other ways to construct a phase space. The use of derivatives, whereby the coordinates of the phase space are successively ordered higher derivatives instead of discrete time shifts, is an alternative. In fact, this is the underlying intuitive concept of the method of delays, but it is not recommended in practice, except for perhaps extremely clean data, since differentiation amplifies noise. A better alternative is the method suggested by Broomhead and King (1986), whereby the Karhunen–Loeve principal value decomposition is applied to the vector time series in Eq. (1). The procedure, called singular-spectrum analysis

IMPLEMENTATION OF PLTMG VERSION 6.0

ON SCRI4D VAX WITH IRIS 4000 GRAPHICS†

By

Jian Yu
Mathematics Department
Florida State University
Tallahassee,FL 32310

1. Introduction

PLTMG is a software package for solving a general second order parameter involved system of elliptic partial differential equations on a two dimensional domain with Dirichlet and/or Neuman boundary conditions. It was originally designed by Bank and his collegues(1979b, 1982b, 1985b, 1988, 1990a) as a tentative program to study the theoretical and practical aspects of the multigrid iterative method, adaptive grid refinement and error estimation procedures, and their interation. The adaptive procedure embodied in PLTMG is based on the ideas of Babuska (1986a, 1986b, 1986c), Rheinboldt, and their co-worker (1986, 1987). The resulting systems of nonlinear equations are solved by a combination of the approximate Newton iteration (See Bank and Rose 1981, 1982) and the hierarchical basis multigrid iteration (See Bank,Dupont and Yserentant, 1988, Yserentant, 1985, 1986a, 1986b, 1986c). The mesh refinement algorithms and data structures used in PLTMG originally followed the ideas of Bank, Sherman and Weiser (1983). The *a posteriori* error estimation procedure used for the adaptive mesh refinement was developed by Bank and Weiser (1986, 1985, 1981). The pseudo-arclength continuation procedure of PLTMG results from the joint work of Bank and Chan (1986). The sparse Gaussian elimination procedure and global stiff matrix storage used for the resulting linear sparse system of solutions was the joint work of Bank and Smith (1987). The deflation technique used for dealing with instability caused by singular matrix was developed by Chan (1984).

The purpose of this report is to present the implementaion of the software package PLTMG Edition 6.0 on the Florida State University (FSU) SCRI4D Vax and to present some examples of PLTMG run on the SCRI4D Vax with the IRIS 4000 color graphics system. The main algorithms embodied in the subroutine PLTMG are explained in detail along with the relevant theoretical background and major results. Data structures used in PLTMG are also illustrated.

† This research is also supported by the Supercomputer Computations Research Institute at Florida State University through contract No. DE-FC05-85ER250000 and by the Geophysical Fluid Dynamics Institute through Grant No. AFOSR 89-0462 and AFOSR 90-0009

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The Role of Potential Vorticity Forcing in Topographically Induced Instabilities

by

Terrence R. Nathan
Atmospheric Science Program
University of California, Davis, CA 95616
U.S.A.

Submitted to
Trends in Atmospheric Sciences
February 1992
Accepted May 1992

Abstract

A quasigeostrophic, two-layer, midlatitude β-plane channel model is used to explore the effects of steady, zonally asymmetric potential vorticity (PV) forcing on baroclinic topographic instability. It is shown analytically that the alteration of the zonal mean flow produced by the interaction of a *resonant* wave with the PV forcing may enhance, suppress or catalyze the topographic instability depending on the *phase* of the PV forcing with respect to the topography. For PV forcing that is maximized upstream (downstream) of the topography, an otherwise stable (unstable) flow can be destabilized (stabilized). If the PV forcing is considered analogous to diabatic heating in a continuous atmosphere, land-sea heating contrasts can be expected to play an important role in the genesis and nonlinear evolution of topographically induced instabilities at middle latitudes.

The weakly nonlinear evolution of the system and the combined effects of topographic and zonally asymmetric PV forcing on intraseasonal oscillations at middle latitudes are briefly discussed.

Introduction

The circulation of the atmosphere is subjected to a vast array of mechanical and thermal forcing mechanisms that span a wide range of space and time scales. For the large-scale motions in the atmosphere, the predominant mechanical and (external) thermal forcing mechanisms are, respectively, topography and land-sea heating contrasts. Such forcing mechanisms have long been recognized as playing an important role in a variety of atmospheric phenomena, ranging from the preferred regional development of the middle latitude cyclone waves to the global heat and momentum balance.

In recent years several studies have demonstrated the importance of large-scale topographic forcing in generating and sustaining low frequency motions in numerical models (e.g., Legras and Ghil 1985; Nathan 1988; Nathan 1989; Mak 1989; O'Brien and Branscome 1990; Tribbia and Ghil 1990; Howell and Nathan 1990; Chou and Loesch 1990) and in rotating annulus experiments (Li et al. 1986; Pfeffer et al. 1989; Bernardet et al. 1990). Although it has been recognized since the work of Charney and Eliassen (1949) that the standing waves at middle latitudes owe their existence in large part to the zonally asymmetric mechanical forcing produced by the earth's topography, it was Charney and DeVore (1979) who established, using a barotropic model, that topographic forcing could yield nonpropagating, exponentially amplifying solutions. These amplifying solutions have since been referred to as topographic or form drag instabilities. Charney and Devore also demonstrated that at finite amplitude these instabilities may lead to a multiplicity of equilibrium flow configurations for a given set of model parameters. One of the equilibrium flows corresponds to a high amplitude wave state with a relatively weak zonal current, and thus has been associated with anomalous flows in the atmosphere. Subsequent studies also have demonstrated the existence of multiple equilibria - and

more generally multiple regimes - in simplified barotropic and baroclinic models of the atmosphere (e.g., Charney and Straus 1981; Pedlosky 1981; Legras and Ghil 1985).

However, because topographically unstable modes and multiple equilibrium states require near resonant flow conditions for their existence, a condition that may be difficult to satisfy for realistic zonal winds in the atmosphere, their importance in the atmosphere has been questioned in recent years (Tung and Rosenthal 1985; Lindzen 1986). Nevertheless, there exists evidence to suggest that topographic instabilities may indeed play an important role in the low frequency variability of the atmosphere. For example, Legras and Ghil (1985) and Tribbia and Ghil (1990) have shown using high resolution numerical models on a sphere that topographic instability may be an integral part of the overall dynamics governing intraseasonal oscillations in the Northern Hemisphere extratropics. To further explain these oscillations, Jin and Ghil (1990) used a midlatitude, β-plane channel model to examine the resonant response of equivalent barotropic flow to topography. They showed that low frequency, finite amplitude oscillations emerge when wave-wave interactions are considered in concert with the topographic form drag. Those oscillations were shown to depend critically on the meridional profile of the zonal mean flow and the existence of a dipole shaped resonance.

Because the importance of topographic instability and multiple equilibria to low frequency variability remains largely unresolved, it is especially important that we determine the robustness of topographic instability to the presence of other internal and external forcing mechanisms. With this in mind, we extend Pedlosky (1981) by examining the role of steady, zonally asymmetric potential vorticity (PV) forcing on baroclinic topographic instability. Such PV forcing can be thought of as mimicking the thermal forcing that arises in a continuous atmosphere due to land-sea heating contrasts. Thus several questions immediately come to mind: Most obviously, can zonally asymmetric PV forcing affect the topographic instability

mechanism? If so, how? How important is the phase between the PV and topographic forcings to the overall dynamics of the system? Are finite amplitude, low frequency oscillations possible?

Model and Governing Equations

To address the questions raised above, the conventional two-layer, quasigeostrophic model is used. Briefly, the model consists of two layers of homogeneous, incompressible fluid of slightly different densities. The system is gravitationally stable, rotating with angular speed Ω about the vertical axis, and confined to a zonally periodic, midlatitude channel of width L. The beta-plane approximation is used to model the effects of the earth's sphericity, i.e., $f = 2\Omega = f_0+\beta' y'$, where $f_0 = 2\Omega \sin\phi_0$ is the Coriolis parameter evaluated at a central latitude ϕ_0, y' the dimensional latitude coordinate, and β' the dimensional, northward gradient of f. The quasi-geostrophic potential vorticity equation can be written in nondimensional form as (Pedlosky 1987)

$$\frac{\partial Q_j}{\partial t} + \beta\frac{\partial \psi_j}{\partial x} + J(\psi_j, Q_j) - (j-1)J(\psi_j, \eta_B) = -r_j\nabla^2\psi_j + (-1)^{j+1}\kappa F(\psi_1-\psi_2) + D_j, \quad (1)$$

where $Q_j = \nabla^2\psi_j+(-1)^j F(\psi_1-\psi_2)$ is the quasigeostrophic potential vorticity and $\psi_j(x,y,t)$ is the geostrophic streamfunction field; the subscript $j = 1$ or 2 denotes variables in the upper or lower layers, respectively; η_B is the bottom topography, F is the internal rotational Froude number, β is the planetary vorticity factor and D_j represents the imposed potential vorticity (PV) sources and sinks. In (1) $\nabla^2 \equiv \partial^2/\partial x^2+\partial^2/\partial y^2$ and $J(a',b') = (\partial a'/\partial x)(\partial b'/\partial y) - (\partial a'/\partial y)(\partial b'/\partial x)$.

Two forms of dissipation are considered in the model: Ekman damping on the upper and lower boundaries, measured, respectively, by the parameters r_1 and r_2, and thickness damping,

Low Frequency Oscillations of Forced Barotropic Flow

by

Terrence R. Nathan
Atmospheric Science Program
University of California
Davis, CA 95616

and

Albert Barcilon
Department of Meteorology and Geophysical Fluid Dynamics Institute
Florida State University
Tallahassee, FL 32306

Submitted to
Journal of the Atmospheric Sciences
Notes and Correspondence
August 21, 1992

Abstract

Jin and Ghil (1990) demonstrate that for topographically resonant flow, low frequency finite-amplitude oscillations may arise from wave-wave interactions and topographic form-drag. Their model is extended to include a zonally asymmetric vorticity source, which is shown to interact with the perturbation field to produce zonally rectified wave fluxes that dramatically alter the Hopf bifurcation from stationary to low frequency oscillations. The frequency, intensity, and general character of these oscillations are shown to depend crucially upon the phasing and relative strength of the forcings.

1. Introduction

Topographic (i.e., form-drag) instability is among the mechanisms proposed as playing an important role in atmospheric low frequency variability (e.g., Frederiksen 1983; Legras and Ghil 1985; Nathan 1988, 1989; Howell and Nathan 1990; Frederiksen and Bell 1987; Jin and Ghil 1990; Tribbia and Ghil 1990; O'Brien and Branscome 1991). As originally shown by Charney and DeVore (1979), alteration of the zonal mean momentum by the topographic form-drag is crucial to topographic instability and the subsequent development of multiple flow regimes at finite amplitude. Therefore, other external forcing mechanisms that interact with the perturbation field to produce changes in the zonal mean momentum will likely play an important role in topographically induced instabilities. This point is underscored by Nathan (1992), who analytically demonstrated within the context of the two-layer baroclinic model that steady, zonally varying potential vortcity (PV) sources can interact with the wave field to alter the zonal mean flow, resulting in dramatic changes in the *linear* topographic instability properties of the system. However, at finite-amplitude only steady state solutions were obtained; the absence of perpetual, low frequency oscillations was attributed to the exclusion of wave-wave interactions within the model.

The goal of this note is to demonstrate the impact of steady, zonally varying vorticity forcing on topographic instability and *finite amplitude*, low frequency oscillations. We use the

quasigeostrophic, barotropic channel model of Jin and Ghil (1990; hereafter JG), who showed that for topographically resonant flow, wave-wave and wave-mean flow interactions are essential in producing Hopf bifurcations from stationary to low frequency oscillations on the intraseasonal time scale. Their results are consistent with those of Tribbia and Ghil (1990), who numerically integrated a fully nonlinear, global barotropic model. In both models the forced wave field was shown to occasionally drift back and forth over the mountain, a feature not found in observations or GCM simulations. JG suggested that the absence of zonally varying thermal forcing may explain this discrepancy. Here we extend JG by introducing a barotropic analogue to this thermal forcing and demonstrate its importance in affecting the phasing and strength of the low frequency oscillation regime.

2. The model and governing equations

The model consists of a viscous, barotropic fluid on a zonally periodic, midlatitude β-plane channel, bounded above by a flat, rigid boundary, below by a bottom topography, and laterally by sidewalls at $y = 0, \pi$. For a steady, meridionally sheared basic flow $\bar{U}(y)$ the nondimensional equations governing the quasi-geostrophic dynamics of a superimposed disturbance, $\psi(x,y,t)$ and mean field, $\bar{\psi}(y,t)$, are:

$$\left(\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right)\nabla^2\psi+(\beta-\bar{U}_{yy})\frac{\partial\psi}{\partial x}+J(\psi,\nabla^2\psi)+J(\psi,h_B)=-\gamma\nabla^2[\psi-\Psi(y)]-\bar{U}\frac{\partial h_B}{\partial x}+q_B, \quad (2.1a)$$

$$\frac{\partial}{\partial t}\frac{\partial^2\bar{\psi}}{\partial y^2}=-\gamma\frac{\partial^2\bar{\psi}}{\partial y^2}-\frac{\partial}{\partial y}\left(\overline{\frac{\partial\psi}{\partial x}h_B}\right)-\frac{\partial}{\partial y}\left(\overline{\frac{\partial\psi}{\partial x}\nabla^2\psi}\right)+\gamma\frac{\partial^2\Psi}{\partial y^2}, \quad (2.1b)$$

with $\psi_x = \bar{\psi}_{yt} = 0$ at $y = 0, \pi$. In these equations $h_B(x,y)$ is the bottom topography, $q_B(x,y)$ is a steady, *zonally varying* vorticity source, $\Psi(y)$ is a perturbation zonal momentum driving, and γ measures the Ekman pumping strength on the upper and lower boundaries.

Equation (2.1b) indicates that changes in the mean flow vorticity are due to damping,